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Transition Control of Instability Waves over an Acoustically Excited Flexible Surface

L. Maestrello*
NASA Langley Research Center, Hampton, Virginia 23665
and
F. W. Grosveld†
Lockheed Engineering & Sciences Company, Hampton, Virginia 23666

Experimental results are presented that demonstrate the coupling of a laminar boundary-layer flow with a typical flexible aircraft panel. It is shown that the boundary layer induces plate oscillations that, in turn, perturb the flow at the same frequencies. This feedback mechanism is an inherent property of laminar boundary-layer flow passing over a flexible plate. As a result, the flexibility of the plate becomes a source of early transition. The laminar boundary layer at the leading edge of the plate reacts to small unsteady disturbances from upstream due to a streamwise pressure gradient. The experiments demonstrate that a nominal sound pressure incident at the leading edge triggers early transition. It is shown that this transition can be delayed by activating a heat source at the leading edge of the plate, which results in downstream cooling.

Introduction

THE coupling of flexible surface oscillations with the boundary layer as a means of controlling skin frictional drag has been investigated since the 1960s by Landahl,[1] Kramer,[2] Wehrmann,[3] Gyorgyfalvy,[4] Laufer and Maestrello,[5] and by others, both in air and in water. The problem has recently been worked by Breuer et al.[6]

In the late 1950s and early 1960s Van Driest and Boison,[7] Wisniewski and Jack,[8] and Reshotko[9] found that the transition Reynolds number increases with a decrease in wall temperature. A transition Reynolds number increasing with cooling is in agreement with the prediction by Lin's linear stability theory for the first mode in compressible flow.[10] However, Mack[11] shows that cooling does not stabilize the highest inviscid modes. McCroskey[12] used localized surface heating at the leading edge of a plate to delay transition at low speeds. Liepmann and Nosenchuck[13] used a feedback mechanism to demonstrate the delay of the transition in water. An analysis of this problem by the method of matched asymptotes as a triple deck was reported by Maestrello and Ting.[14] Similar studies were conducted experimentally in air by Maestrello[15] and Struminski et al.,[16] and numerically in water by Kral.[17] Recently, the experimental technique has been expanded from delaying transition[18] to controlling turbulence over a plate.[19] Numerically, transition delay over flat and curved surfaces has been studied and extended to control separated flow over an airfoil.[20-22] Research into the problem of viscous drag reduction in boundary layers has been summarized in a 1989 publication edited by Bushnell and Hefner.[23]

Instability waves leading to the transition process have been observed to be the result of traveling Tollmien-Schlichting waves imposed on the freestream velocity by the vibration of the leading edge of the plate.[24,25] In addition, acoustic waves are known to be transformed into Tollmien-Schlichting waves at a rigid leading edge. These receptivity problems, a term suggested by Obremski et al.,[26] have been investigated by Tam,[27] Kachanov et al.,[28] Goldstein,[29] Lowson,[30] Bechert,[31] Singurson and Roshko,[32] Domaradski and Metcalfe,[33] Carpenter,[34] Yeo and Dowling,[35] Kerschen,[36] Shapiro,[37] Gapanov,[38] and others. In these studies, the path to transition has been by a single wave interacting with itself causing distortion or interaction with other single waves causing resonances. Comprehensive investigations of different types of excitations that will generate Tollmien-Schlichting waves are needed to understand the practical applications with regard to boundary-layer control. One of these is the laminar boundary layer over a flexible plate. A larger number of instability modes is present in a laminar boundary layer over a flexible surface than in a laminar boundary layer over a rigid surface because of the coupling of the panel vibration and the boundary layer. Acoustically induced disturbances from upstream accelerate the laminar boundary layer into transition by forcing the flexible plate to respond with a relatively high amplitude and with a large number of resonance modes. Localized heating at the leading edge alters the growth of the instabilities due to progressive cooling downstream.[19,39] As a result, the flow stability is increased by the modifications of the velocity and temperature profiles along the boundary layer. The present work indicates that plate flexibility can induce early transition and shows the combined effects of an incident acoustic wave and surface heating near the leading edge on the boundary layer along a flexible surface.

The purpose of this paper is to report on three related boundary-layer experiments with a rigid surface containing a flexible panel. These experiments consider 1) the instability growth in a laminar boundary layer due to plate vibration, 2) the amplification of instability waves in a laminar boundary layer due to upstream sound, and 3) the delay of transition due to localized upstream surface heating.

Flow Validation

The experiments were conducted in an open-circuit wind tunnel with a 38.1 × 38.1-cm test section and a maximum speed of 36.6 m/s. The 38.1 × 279.4 × 2.54-cm rigid test plate features an elliptical leading edge. The center part of the plate has a section cut out to enable the installation of a 30.5 × 20.3 × 0.10-cm flexible aluminum plate with clamped edges (see Fig. 1). Note that the bottom side of the flexible plate is covered by a streamlined rigid surface so that the plate

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*Scientist, Structural Acoustics Branch, Acoustics Division, Mail Stop 463. Associate Fellow AIAA.

†Section Manager, Aeroacoustics and Structural Acoustics, Langley Program Office, 144 Research Drive. Associate Fellow AIAA.

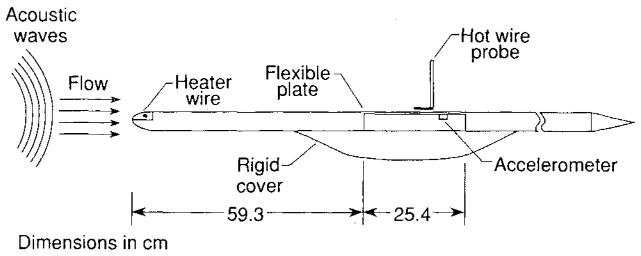
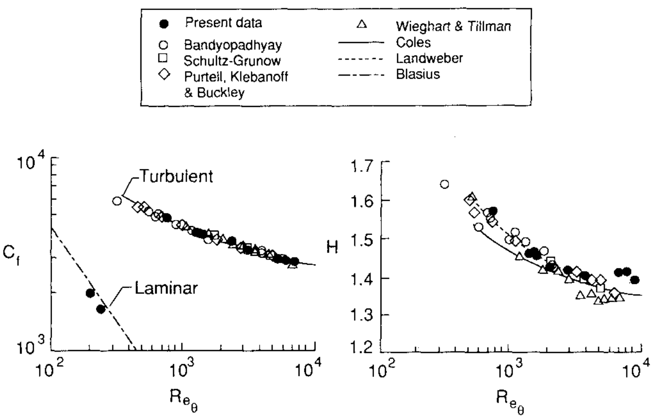
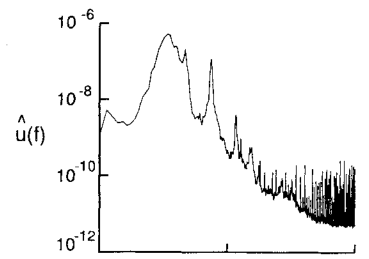


Fig. 1 Test configuration of the flexible plate mounted in the rigid model.

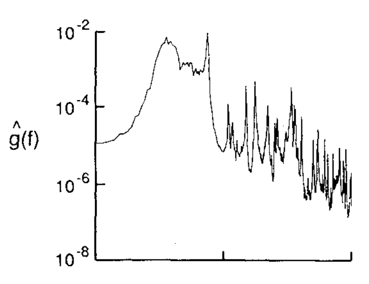


a) Skin friction coefficient **b) Shape factor**

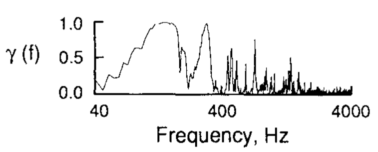
Fig. 2 Experimental validation.



a) Normalized velocity perturbation



b) Normalized panel acceleration



c) Coherence

Fig. 3 Coupling between laminar boundary layer and panel vibration.

is exposed to the flow only at its upper surface. Flow control is accomplished using a thin, electrically heated nickel-chrome wire embedded in a substrate (Space Shuttle tile) at the leading edge. The heater wire can be operated in both ac and/or dc modes. A hot wire, an accelerometer, a microphone, and surface temperature sensors were used to measure the response of the flow and the flexible plate and the wall temperature. The hot wire has a length-to-diameter ratio of 300, is fabricated from 0.0125-mm-diam platinum-rhodium wire, and is located at the middle of the centerline 0.51 cm above the flexible plate. The vertical distance from the plate corresponds to γ/δ of 0.974 for the laminar boundary layer, 0.925 for the laminarized, and 0.393 for the turbulent boundary layer. The accelerometer has a mass of 0.3 g and is positioned on the centerline, 7.62 cm from the end of the flexible plate, to measure contributions from the even as well as the odd plate modes. The 0.64-cm-diam condenser microphone is of the pressure response type and is mounted on the sidewall of the tunnel above the leading edge of the rigid plate. A square signal is amplified and fed to a loudspeaker, which is placed well ahead of the leading edge. The incident acoustic wave has a fundamental frequency $f = 286$ Hz, corresponding to an oscillating mode of the flexible plate.

The initial measurements involved flow validation over the plate for various Reynolds numbers Re_θ (based on momentum thickness) in terms of the skin friction coefficient C_f and the shape factor H. It was established that turbulent boundary-layer flow from the wall did not affect the boundary-layer flow over the flexible plate. The results are shown in Fig. 2, and comparison with experimental data from Refs. 40 and 41 shows good correlations of C_f and H as function of Re_θ.

Boundary-Layer Experiments

Laminar Boundary Layer Over a Flexible Plate

The unstable behavior of the laminar boundary layer over a flexible surface is maintained by the motion of the surface. This is markedly different from the stable nature of the boundary layer over a rigid surface. The coupling between the plate vibration and the laminar boundary layer enhances the transition process. Shown in Fig. 3 are the normalized power spectral densities of the velocity fluctuation $\hat{u}(f)$, the plate acceleration $\hat{g}(f)$, and the coherence function $\gamma(f)$ between the velocity and the acceleration measured by the hot-wire anemometer and accelerometer shown in Fig. 1. The Reynolds number and the shape factor have values of $Re_\theta = 756$ and $H = 2.43$, indicating a laminar boundary layer. The velocity and the acceleration spectra have similar amplitude vs frequency, indicating unique experimental evidence of coupling between the plate vibration and the boundary layer. In a laminar boundary layer, surface vibrations impose broadband velocity fluctuations on the layer with the same spatial and temporal scales as the surface motion. The displacement of the flexible plate, although small, is sufficient to impart a permanent imprint onto the receptive shear layer. Thus, the flexible surface can accelerate the transition process of the laminar boundary layer

Transition Induced by an Upstream Acoustic Wave

In the second experiment, an acoustic square wave ($f = 286$ Hz) is imposed on the upstream mean flow by a loudspeaker, resulting in an overall sound pressure level of 106 dB measured at the leading edge. The spectrum of the wall pressure fluctuations is shown in Fig. 4. Discrete spikes are superimposed on the random fluctuations from the wall turbulent boundary layer. The first spike corresponds to the frequency of the square wave input (286 Hz), whereas the other spikes in the spectrum represent its Fourier components. The boundary layer near the leading edge of the plate is highly receptive to an acoustic wave upstream. The flow and sound couple at the leading edge and, as a result, the transition location moves upstream on the flexible plate, resulting in turbulent flow at the anemometer and accelerometer measurement locations

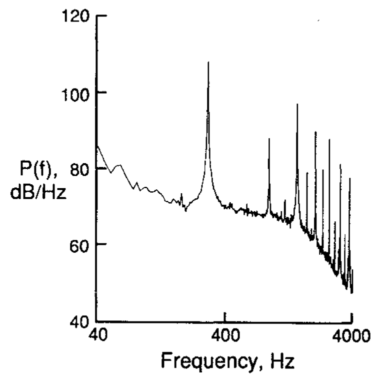


Fig. 4 Power spectral density of the wall pressure fluctuations above the leading edge in the presence of upstream sound.

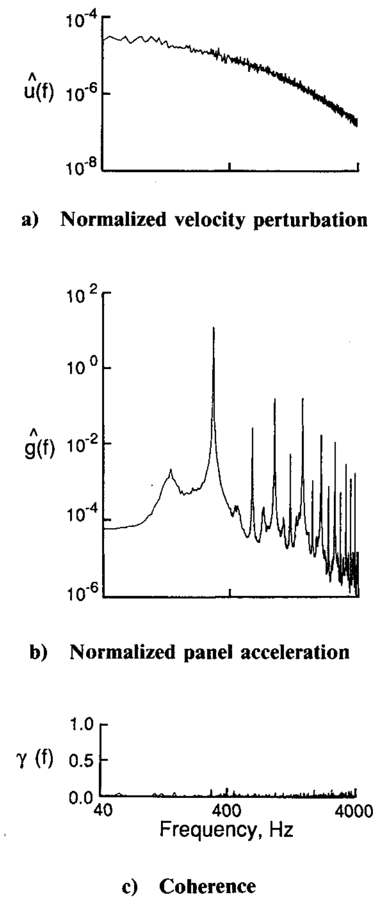


Fig. 5 Coupling between turbulent boundary layer and panel vibration.

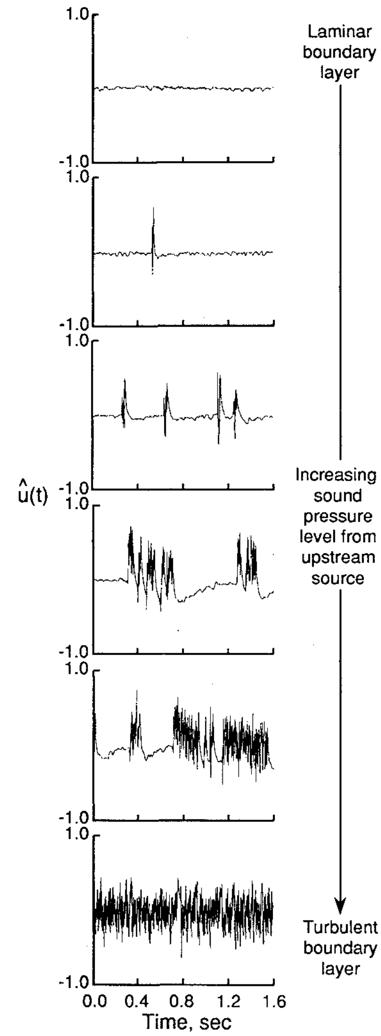


Fig. 6 Normalized perturbation velocity between laminar and turbulent states as a result of increasing sound pressure level.

(Fig. 1). The Reynolds number and the shape factor at the anemometer measurement location (Fig. 1) change to $Re_\theta = 1890$ and $H = 1.40$, respectively, indicating a turbulent state. An incident sound wave with a much higher amplitude (at least twice[18]) is needed to induce early transition at the same location on a rigid plate, indicating that the flexible surface accelerates the transition. Figure 5 shows the normalized power spectral density of the turbulent boundary-layer velocity fluctuations and the panel acceleration as well as their incoherence.

Comparing the turbulent flow results (Fig. 5) with the laminar flow results in Fig. 3, the power spectral density of the velocity fluctuations in the turbulent boundary layer is two orders of magnitude higher than in the laminar layer. Also, the power spectral density of the plate acceleration increases three orders of magnitude with distinct spikes matching the frequencies of excitation. The spectrum of the velocity fluctuations in Fig. 5 is smooth, typical for a turbulent boundary layer, whereas the spectrum of the plate acceleration is not. Note that the coherence function is nearly zero, in contrast with the laminar boundary-layer case, which is strongly coherent. One possible explanation is that random phase cancellation takes place between the flow and the plate oscillations when there is no time delay.

The sequence of time histories of the normalized velocity perturbation $\hat{u}(t)$ from the laminar to turbulent state as a result of the gradual increase in sound pressure level (60–106 dB) on the wall at he leading edge of the rigid plate is shown in Fig. 6. Disturbances, each with its own frequency content and amplitude, are shown in each stage of transition as the transition location moves upstream and passes through the measuring station on the flexible plate.

Transition can be triggered earlier by an acoustic wave of weaker intensity in the presence of a flexible surface than in the presence of a rigid one. The amplitude of oscillation of the laminar boundary layer due to the presence of the flexible surface is an order of magnitude higher than over a rigid surface at the same Reynolds number. The significant increment in oscillation amplitude at the instability frequencies and at the excitation frequencies contributes to accelerated transition and an increase in skin frictional drag.

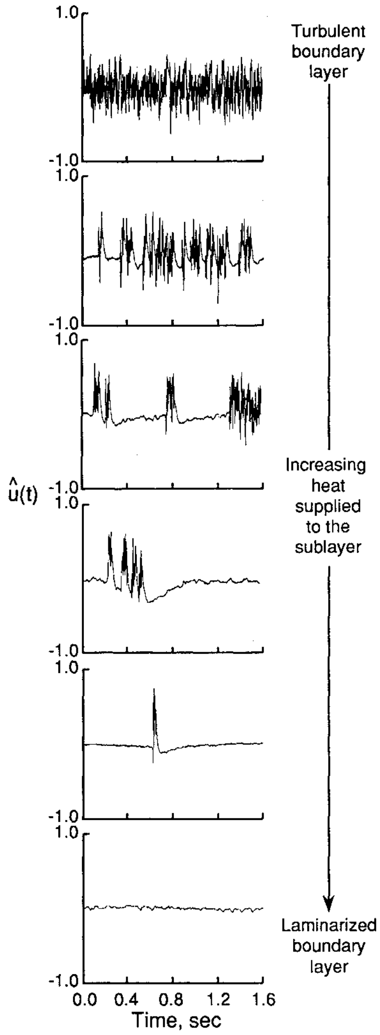


Fig. 7 Normalized perturbation velocity between turbulent and laminar states as a result of increasing heat supply at the leading edge.

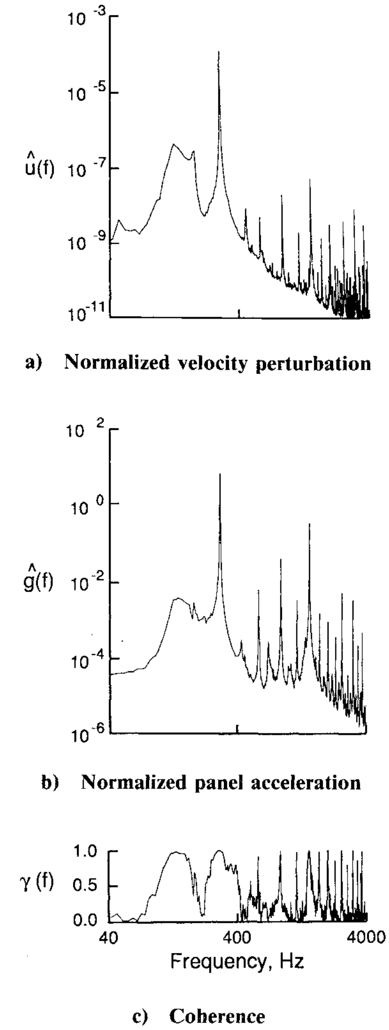


Fig. 8 Coupling between laminar boundary layer and panel vibration in the presence of upstream sound.

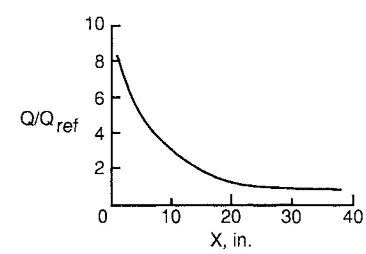


Fig. 9 Rate of spatial cooling downstream of the heating elements.

Transition Control by Localized Surface Heating

Control of transition is achieved by localized leading-edge surface heating, the configuration of which is illustrated in Fig. 1. Because of the favorable pressure gradient at the leading edge, heat released by the surface wire couples with the sublayer producing a temperature gradient in the downstream direction. Figure 7 shows the normalized perturbation velocity $\hat{u}(t)$ as it changes from the turbulent to the laminar state. Distinct features in the time sequences of the perturbations are shown in each stage as the transition shifts downstream past the measurement station. The heat flux through the wire is gradually increased until laminar behavior is established. Figure 8 shows the normalized velocity perturbation power spectral density in the laminar boundary layer after the transition delay, the normalized panel acceleration power spectral density, and their mutual coherence. The presence of the upstream acoustic wave is apparent by the distinct peaks in the velocity and acceleration spectra at which frequencies the coherence $\gamma(f)$ is close or equal to 1. The fact that sound from upstream is present in the boundary layer before, during, and after the transition delay indicates that the thermal gradient in the sublayer along the direction of flow, which changes the viscosity, is the stabilizing mechanism. Figure 9 shows the rate of decrease of the spatial cooling parameter $Q = [(T - T_e)/T_e]/(P/K_eT_e)$, normalized to the unheated state Q_{ref} as a function of distance X. P is the input power to the heater, K_e the thermal conductivity, and T and T_e the local and freestream total temperatures of the flow. The cooling parameter Q was obtained from temperature measurements over an insulated surface along the direction of the flow. The boundary-layer thickness δ, skin friction coefficient C_f, shape factor H, Reynolds number Re_θ, boundary-layer momentum thickness θ, power required by the heater $P = IE$, and the power reduction in the flow D_fU_e are shown in Table 1 for the laminar and turbulent states of the boundary layer. I is the current, E the voltage, and D_f the skin frictional drag. Table 1 shows that the input power to the heater is higher than the power saved by the reduction in skin friction drag. The power required by an acoustic field to change the transition location is significantly less than the power lost by skin frictional drag. Because of the unstable transition condition, the acoustic power required to trip the boundary layer as the transition point shifts forward is much less than the power required to re-

Table 1 Mean flow parameters, power required by the heater and power reduction in the flow for different states of the boundary layer with freestream velocity U_e = 18.4 m/s

State of flow	C_f	R_θ	H	δ, cm	θ, cm	$P = IE$, W	$D_f U_e$, W	Remarks
Laminar	0.00085	756	2.43	0.48	0.063	0	0	Natural state
Turbulent	0.00470	1890	1.42	1.19	0.147	0	0	Turbulent state induced by sound
Laminar	0.00070	790	2.50	0.50	0.058	129	54	Transition delayed by heating

establish the transition point to its original location by surface heating. No attempt has been made to reduce the power needed by the heating system by improving its efficiency.

At high supersonic speeds, localized aerodynamic heating is a natural phenomenon, which can be utilized to enhance the stability of the boundary layer.[35] The stabilizing effect in the critical layer near the wall is due to the dependence of viscosity on temperature. Viscosity decreases with temperature; thus, the dissipation is due to the gradual cooling along the direction of flow. By stirring up the critical thermal layer, the stabilizing influences diminish quickly. Future experiments are anticipated in which these techniques will be used not only to control transition but also to control turbulence. In summary, the response of a flexible structure is altered by the presence of sound, temperature, pressure gradient, and turbulence.

Conclusions

An important source of laminar boundary-layer instability has been identified for flow over a flexible plate. The instability is associated with the coupling between the plate vibration and the laminar boundary layer. The perturbations imparted to the laminar layer by the plate vibration are broadband in nature, unlike the leading-edge instability waves that are spawned from a single mode. Flow perturbations induced by the plate oscillations develop into unstable Tollmien-Schlichting waves, which shows that the laminar boundary layer is very receptive to small structural vibrations of the surface.

Results show clear evidence that flow-structure-sound interactions cannot be simulated by the structure and the flow by themselves or by the structure and the sound alone, but that they are all inherently coupled, both spatially and temporally.

The experiments demonstrate that the transition over a typical aircraft panel in the presence of an acoustic field can be controlled by localized surface heating at the leading edge, thus reducing the skin friction drag over the panel surface.

The present experiments broaden the field of flow control phenomena beyond instability wave phenomena over a rigid surface. Both sound pressure level and structural vibration were found to be involved in the destabilization of the laminar boundary layer.

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